UNIT-III

Transient response of DC and AC circuits :-

[i] Transient response of D.C circuits.

When Excitation is D.C supply { For R-L Series circuit, R-C Seriec circuit, R-L-C Series circuit

[ii] Transient response of A.C circuits.

(i) R-L Series circuit
(ii) R-C Series circuit
(iii) R-L-C Series circuit

} Excitation is A.C supply.

* ~~The Voltage and current chan~~

~~Transient response~~ Study State :- A circuit having Constent sources and is said to be in Study state if the current and Voltages donot change with time, Hence circuits with Currents and Voltages having (Constent amplitude & Constent frequency Sinosodial function are Considerd to be in Study State.

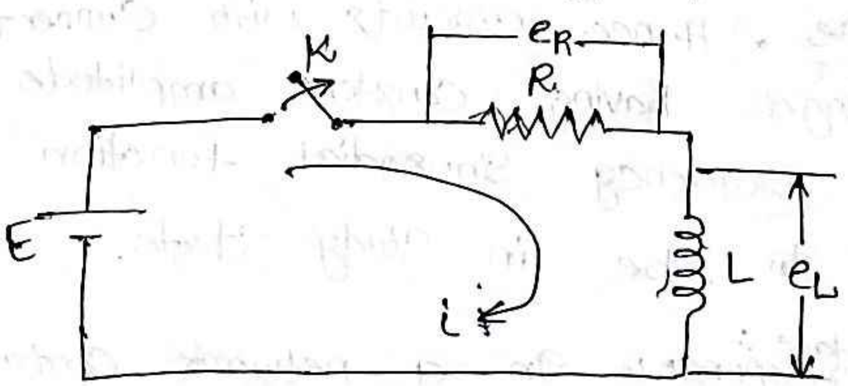
Transient ~~response~~ State :- In a network containing energy Storage elements with change in excitation the currents and Voltages change from 1 state to another state. The behaviour of the Voltage and current

When it is changed from 1 state to the another State is called as transient State.

transient Time:- The time taken for the circuits to changed from one Study state to the another Study State is called transient time.

Element	Voltage Reaction	Current Reaction
Resistor	$\boxed{V = i \cdot R}$	$i = \frac{V}{R}$
Inductor	$\boxed{V = L \cdot \frac{di}{dt}}$	$i = \frac{1}{L}\int v \cdot dt$
Capacitor	$V = \frac{1}{C}\int i \cdot dt$	$i = C \cdot \frac{dv}{dt}$

* Transient Response of Series R-L circuit. In DC Excitation



Consider a Series RL circuit With Resistance (R) Ω and inductance (L) Henery which is Supplied by DC Voltage (E).

Let Voltage across the resistor

is given by e_R

So,

$$\boxed{e_R = i \cdot R} \text{ --- (i)}$$

and let Votage across the inductor

e_L is given by

$$\boxed{e_L = L \cdot \frac{di}{dt}} \text{ --- (ii)}$$

By applying KVL for the given circuit,

$$E = e_R + e_L$$

So, from (i) & (ii)

$$\boxed{E = i \cdot R + L \cdot \frac{di}{dt}} \text{ -- (iii)}$$

By apply laplace transform on both side.

$$\frac{E}{S} = R \cdot I(S) + L \cdot [S I(S) - i(0)]$$

Where $i(0)$ is Current ~~fo ftto~~ flowing through the circuit when the switch K is open, $i(0) = 0$

$$R \cdot I(S) + L(S I(S)) = \frac{E}{S}$$

$$I(S)[R + LS] = \frac{E}{S}$$

$$I(S) = \frac{\frac{E}{S}}{(R+LS)}$$

$$I(S) = \frac{E}{S(R+LS)}$$

$$I(S) = \frac{E}{L \cdot S\left(S+\frac{R}{L}\right)}$$

$$\boxed{I(S) = \frac{E/L}{S\left(S+\frac{R}{L}\right)}}$$

By partial friction

$$I(S) = \frac{E/L}{S\left(S+\frac{R}{L}\right)} = \frac{A}{S} + \frac{B}{\left(S+\frac{R}{L}\right)}$$

$$E/L = A\left(S+\frac{R}{L}\right) + BS \quad \text{--- (iv)}$$

$$S = 0$$

$$\frac{E}{L} = A \cdot \left(0+\frac{R}{L}\right)$$

$$\frac{E}{L} = A \cdot \frac{R}{L}$$

$$\boxed{A = \frac{E}{R}}$$

Put $S = -\frac{R}{L}$ in equ (iv)

$$\frac{E}{L} = A\left(-\frac{R}{L}+\frac{R}{L}\right) + B\left(-\frac{R}{L}\right)$$

$$\frac{E}{L} = A(0) + B\left(-\frac{R}{L}\right)$$

$$\frac{E}{\cancel{L}} = -B \cdot \frac{R}{\cancel{L}}$$

$$\boxed{B = -\frac{E}{R}}$$

$$I(S) = \frac{E/R}{S} + \frac{(-E/R)}{(S + R/L)}$$

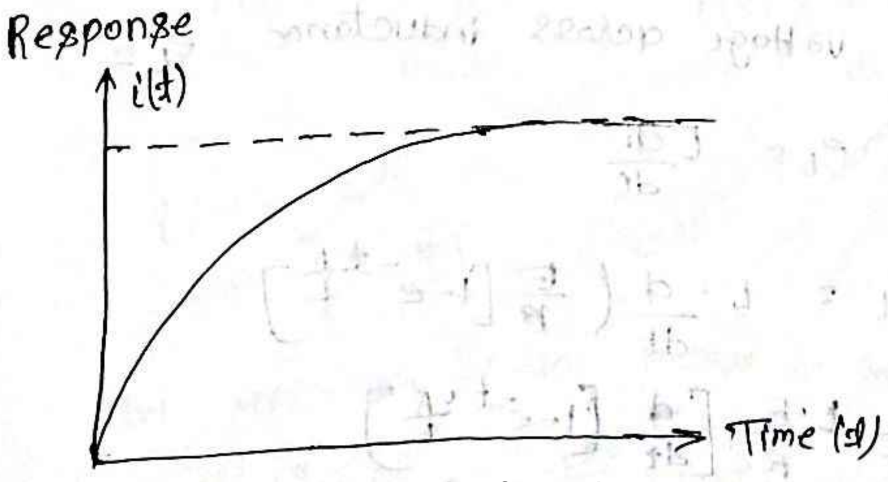
$$I(S) = \frac{E/R}{S} - \frac{E/R}{\left(S + \frac{R}{L}\right)}$$

By applying laplace inverse,

$$L^{-1}\left[I(S)\right] = L^{-1}\left[\frac{E/R}{S}\right] - L^{-1}\left[\frac{E/R}{S + R/L}\right]$$

$$i(t) = \frac{E}{R}\left[L^{-1}\left(\frac{1}{S}\right) - L^{-1}\left[\frac{1}{S + R/L}\right]\right] \qquad \boxed{L^{-1}\left(\frac{1}{S}\right) = 1}$$

$$\boxed{i(t) = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]} \quad \text{— (iv)}$$



Put $t = \frac{L}{R}$ in equ (iv)

$$i(t) = \frac{E}{R}\left[1 - e^{-\frac{L}{R}\times\frac{R}{L}}\right]$$

$$= \frac{E}{R}\left[1 - e^{-1}\right] = \frac{E}{R} \times 0.632$$

$i(t) = 63.2\%$ of $\frac{E}{R}$

$i(t) = 63.2\%$ of Steady state value.

* Time constent of RL circuit :- It is defined as the time during which the current increases to 63.2% of its steady state value.

It is denoted by 'τ' (Tou) ~~to~~

$$\cancel{\tau} = \frac{L}{R} \qquad \boxed{\tau = \frac{L}{R}}$$

* Transient voltage across the Resistor e_R :-

$$e_R = i \cdot R$$

where $i = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$

$$e_R = E\left[1 - e^{-\frac{R}{L}t}\right]$$

transient voltage across inductance e_L =

$$e_L = L\frac{di}{dt}$$

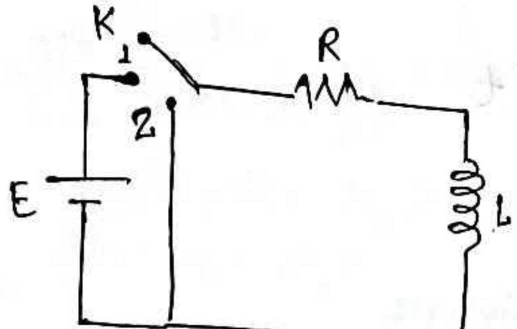
$$e_L = L \cdot \frac{d}{dt}\left(\frac{E}{R}\left[1 - e^{-t\frac{R}{L}}\right]\right)$$

$$e_L = \frac{L \cdot E}{R}\left[\frac{d}{dt}\left[1 - e^{-t \cdot \frac{R}{L}}\right]\right]$$

$$i = \frac{E}{R}\left[e^{-t\cdot\frac{R}{L}} \cdot \frac{R}{L}\right]$$

$$e_L = E\left[e^{-t\cdot\frac{R}{L}}\right]$$

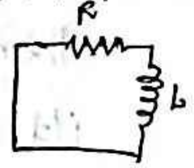
* RL dacay Transient :- Consider the Circuit with Series RL element by using multiple Switch



By Applying KVL

$$iR + L\cdot\frac{di}{dt} = 0$$

joint with position 2



By applying laplace Transformation.

$$i(S)\cdot R + L\left[S\cdot I(S) - i(0)\right] = 0$$

$$\boxed{i(0) = \frac{E}{R}}$$

RL decay में $\frac{E}{R}$ use करता है।

$$i(S)\cdot R + L\cdot SI(S) - L\cdot\frac{E}{R} = 0$$

$$\boxed{i(S)R + L\cdot SI(S) = L\cdot\frac{E}{R}}$$

$$i(S)\left[R + LS\right] = \frac{L\cdot E}{R}$$

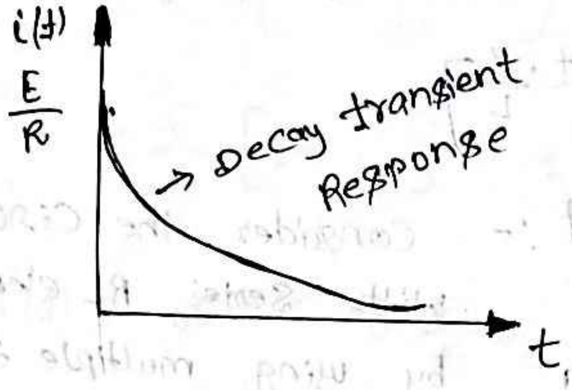
$$i(S) = \frac{L\cdot\frac{E}{R}}{R+LS} \quad ; \quad i(S) = \frac{\cancel{L}\cdot E/R}{\cancel{L}\left[S+\frac{R}{L}\right]} =$$

$$i(S) = \frac{E/R}{\left(S+\frac{R}{L}\right)}$$

By applying laplace inverse on both side.

$$L^{-1}\left[I(S)\right] = L^{-1}\left[\frac{E/R}{\left(S+\frac{R}{L}\right)}\right]$$

$$\boxed{i(t) = \frac{E}{R}\, e^{-t\,R/L}}$$



When

time $(t) = \frac{L}{R}$

then

$$i(t) = \frac{E}{R}\, e^{-\frac{L}{R} \times \frac{R}{L}}$$

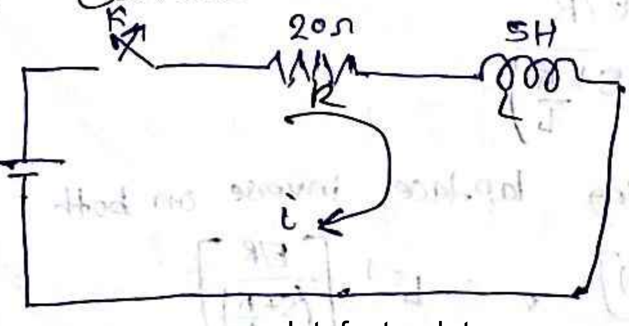
$$i(t) = \frac{E}{R}\, e^{(-1)}$$

$$\boxed{\cancel{i(t) =} \frac{E}{R}} \qquad i(t) = \frac{E}{R}\,(0.367)$$

So, decay transient = 36.7% of E/R

Time Constant :- Time Constant is defined as the time required to reach 36.7% of its initial Value

For the given circuit shown in fig calculate the total current flowing through the circuits



By applying KVL for the circuit.

$20i+5\left[\frac{di}{dt}\right]=10$; By applying laplace transform on both side,

$20I(S)+5\left(SI(S)-i(0)\right)=\frac{10}{S}$

$i(0)=0$

$I(S)\left[20+5S\right]=\frac{10}{S}$; $I(S)=\frac{10}{S(20+5S)}$

$I(S)=\frac{10}{S\times 5\left(S+\frac{20}{5}\right)}=\frac{\frac{10}{5}}{S(S+4)}=\frac{2}{S(S+4)}$

$I(S)=\frac{2}{S(S+4)}$ Applying partial fractions

$2=A(S+4)+B(S)$ $S=0$

$I_S=\frac{A}{S}+\frac{B}{(S+4)}$ $2=A4+0$ $A=1/2$

put $S=-4=$ 2

$I_S=\frac{1/2}{S}-\frac{1/2}{(S+4)}$

Applying inverse laplace transform on both side

$i(t)=\frac{1}{2}\left[1-e^{-4t}\right]$

*A 100 V dc is supplied to series RL circuits with resistance $R=25\Omega$ calculate the total ~~current~~ in the circuit when having Double the time constant.

⇒ Given data :- (Series R-L Circuit):-

$E=100V$

$R=25\Omega$

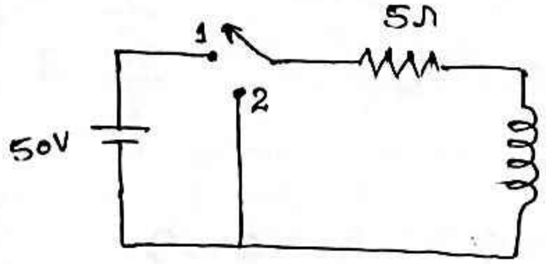
$\tau'=2\tau=2\times\frac{L}{R}$

$i(t)=\frac{E}{R}\left(1-e^{-t\frac{R}{L}}\right)$

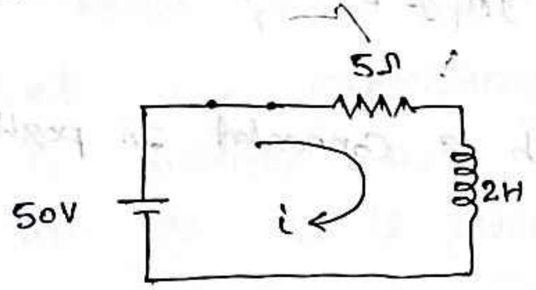
$i(t)=\frac{100}{25}\left(1-e^{-2\times\frac{L}{R}\times\frac{R}{L}}\right)$

$i(t)=\frac{100}{25}\left(1-e^{-2}\right)=3.458$ Amp

* For the Series R-L circuit shown in fig. calculate current flowing through the circuit when the switch is connected to position 1 & position 2



Cas (i) :- When Switch is connected to position 1



$5i + 2\frac{di}{dt} = 50$

$sI(s) + 2[sI(s)] = 50/s$, $I(s) = \frac{50}{s(s+2.5)} = \frac{50 \cdot 25}{2[s+5/2]s}$

$I(s)[s+2s] = 50/s$

$I(s) = \frac{25}{s+2.5} = \frac{A}{s} + \frac{B}{(s+2.5)}$

$25 = A(s+2.5) + B(s)$, $s=0$, $25 = A(2.5)$, $A = \frac{250}{25} = 10$

$$5i + 2 \cdot \frac{di}{dt} = 50$$

Taking laplace on both sides

$$5I(s) + 2 \cdot [s \cdot I(s) - i(0)] = \frac{50}{s} \qquad i(0) = 0$$

$$I(s)[5+2s] = \frac{50}{s}$$

$$I(s) = \frac{50}{s(5+2s)} = \frac{50}{s \times 2\left(s+\frac{5}{2}\right)} = \frac{25}{s(s+2.5)}$$

$$\frac{25}{s(s+2.5)} = \frac{A}{s} + \frac{B}{(s+2.5)}$$

$$25 = A(s+2.5) + B(s)$$

Put $s=0$

$$25 = (A \cdot 2.5) = A = \frac{250}{25} = 10$$

$s = -2.5$

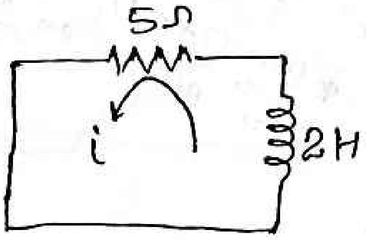
$$25 = B(-2.5)$$

$$B = \frac{-2.5}{25} = -10$$

$$I(s) = \frac{10}{s} - \frac{10}{(s+2.5)}$$

$$i(t) = 10\left(1 - e^{-2.5t}\right)$$

Case (ii):- When the switch is connected to position 2.



Applying KVL

$$5i + 2\frac{di}{dt} = 0$$

Taking Laplance transform on both sides

$$5I(s) + 2\left[sI(s) - i(0)\right] = 0$$

$i(0) = \frac{E}{R}$ in RL decay transient

$\frac{50}{5} = 10$

$$i(0) = 10$$

$$5I(s) + 2 \cdot sI(s) = 20$$

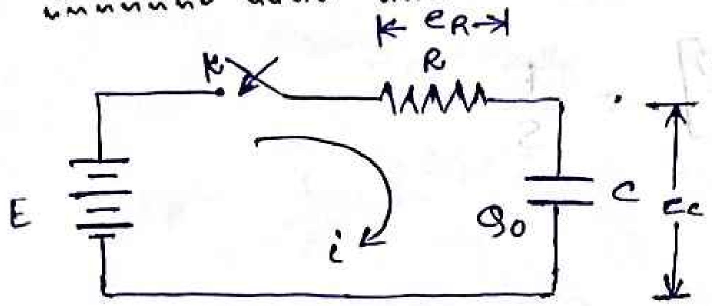
$$I(s) = \frac{20}{(5+2s)}$$

$$I(s) = \cancel{\frac{20}{s\left(2+\frac{5}{12}\right)}} \quad \frac{20}{2(s+2.5)} =$$

$$I(s) = \frac{10}{s+2.5}$$

$$\boxed{i(t) = 10e^{-2.5t}}$$

Series R-C circuit :-



q_0 = initial charge on capacitor

Consider A series R-C circuit and the applied DC Voltage is E which is connected across the switch k.

Let the capacitance has an initial charge of q_0 coulombs then initial voltage across the capacitor V_0 is given by

$$\boxed{V_0 = \frac{q_0}{C}}$$

But for initial voltage across capacitor $\boxed{V_0 = \frac{q_0}{C}} = 0$

By applying KVL for the circuit

$$e_R + e_C = E$$

$$\boxed{iR + \frac{1}{C}\int i\,dt + V_0 = E}$$

Initial Vol across capacitor, $\boxed{V_0 = \frac{q_0}{C}, \quad V_0 = 0}$

Initially there is no charge

then $q_0 = 0 \Rightarrow V_0 = 0$

From eq (i)

$$iR + \frac{1}{C}\int i\,dt + 0 = E$$

$$iR + \frac{1}{C}\int i\,dt = E$$

Taking L.T on both sides,

$$I(S).R + \frac{1}{C}\left[\frac{I(S)}{s}\right] = \frac{E}{S}$$

Series R-C circuit,

$$I(S)\left[R+\frac{1}{C(S)}\right] = \frac{E}{S}$$

$$I(S) = \frac{\frac{E}{S}}{\left(R+\frac{1}{CS}\right)}, \quad I(S) = \frac{E/s}{\left(\frac{RCS+1}{CS}\right)}$$

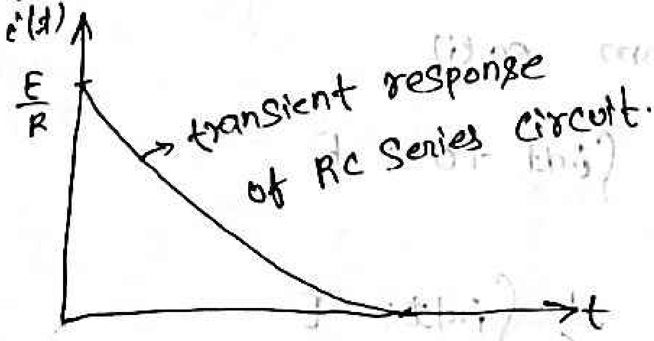
$$I(S) = \frac{\frac{E}{\cancel{S}}}{\frac{RCS+1}{C\cancel{S}}}, \quad \boxed{I(S) = \frac{EC}{RCS+1}}$$

$$I(S) = \frac{E\cancel{C}}{R\cancel{C}\left(S+\frac{1}{RC}\right)}, \quad I(S) = \frac{E/R}{\left(S+\frac{1}{RC}\right)}$$

By applying inverse L.T on both side,

$$i(t) = \frac{E}{R} \cdot e^{-\frac{1}{RC}t}$$

~~By applying~~



Transient voltage across the resistance

$$e_R = I \cdot R$$

$$e_R = \frac{E}{\cancel{R}} \cdot e^{-1/RC \cdot t} \cdot \cancel{R}$$

$$\boxed{e_R = E \cdot e^{-1/RC \cdot t}}$$

Transient voltage across the ~~conductor~~ capacitor.

$$e_c = \frac{1}{C}\int_0 i\,dt$$

$$e_c = \frac{1}{C}\int_0^t \frac{E}{R} \cdot e^{-1/RC \cdot t}\,dt$$

$$e_c = \frac{E}{RC}\int_0^t e^{-1/RC \cdot t}\,dt$$

$$\boxed{e_c = \frac{E}{RC}\left[\frac{e^{-1/RC \cdot t}}{-1/RC}\right]_0^t}$$

$$e_c = \frac{E}{\cancel{RC}}\left[\cancel{RC}\left[-e^{-1/RC\, t}\right]_0^t\right]$$

$$e_c = E\left[-e^{-1/RC\, t}\right]_0^t$$

$$e_c = E\left[-e^{-1/RC\, t} - \left(-e^{-1/RC(0)}\right)\right]$$

$$e_c = E\left[-e^{-1/RC\, t} + 1\right]$$

$$\boxed{e_c = E\left[1 - e^{-1/RC\, t}\right]}$$

Time Constant:- $\boxed{\text{At } (T) = RC}$

$$i(t) = \frac{E}{R} \cdot e^{-\frac{1}{RC} \times RC}$$

$$i(t) = \frac{E}{R} \cdot e^{-1}$$

$$i(t) = \frac{E}{R}(0.367) = 36.8\% \text{ of } \frac{E}{R}$$

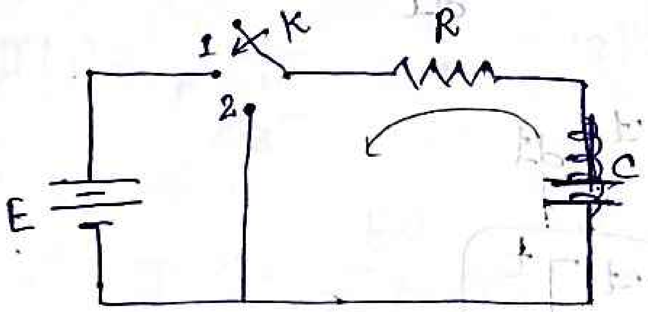
Voltage across capacitor

$V_c + (V_0)$

$\Rightarrow \frac{1}{C}\int i\,dt + E = 0$

$\Rightarrow$

* RC decay transients:-



Consider the circuit with resistor R and the capacitor C are connected in series and the applied dc voltage is E initially the switch has been in position (1) for sufficient time. Before the switch is move to position (2) the capacitor gets charged to the voltage E.

When the switch is move to position (2)

then applying KVL

$$i \cdot R + \frac{1}{C}\int i \cdot dt + E = 0$$

or $$i R + \frac{1}{C}\int i \cdot dt = -E$$

$$I(s)R + \frac{1}{C}\left[\frac{I(s)}{s}\right] = -\frac{E}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = -\frac{E}{s}$$

$$I(S) = \frac{-E}{s\left[R+\frac{1}{Cs}\right]}$$

$$I(s) = \frac{-E}{s \times \left[\frac{RCs+1}{Cs}\right]}$$

$$I(S) = \frac{-E}{s \times \frac{1}{Cs}\left[RCs+1\right]}$$

$$I(S) = \frac{-EC}{RCs+1}$$

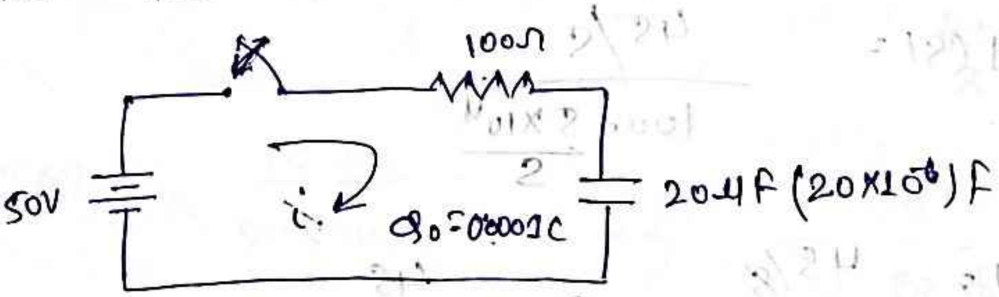
$$I(s) = \frac{-EC}{RC\left(S+\frac{1}{RC}\right)}$$

By applying L.T inverse on both sides.

$$\boxed{i(t) = \frac{-E}{R}\left(e^{-\frac{1}{RC}t}\right)}$$

* The 20 μF capacitor has initial charge of $q_0 = 0.0001$ Coulombs which is connected to in series to 100 Ω. The total circuit is supplied by 50V DC find the transient current.

=>



By appling KVL for the circuit

$$\Rightarrow 100i + \frac{1}{20\times10^{-6}}\int i\,dt + V_0 = 50$$

$$V_0 = \frac{q_0}{C} = \frac{0.0001}{20\times10^{-6}} = 50$$

$$= 100i + \frac{10^6}{20}\int i\,dt = 45$$

$$100\, i(S) + \frac{10^6}{20}\left[\frac{I(S)}{S}\right] = \frac{45}{S}$$

$$\cancel{100\, i(S) +}\ I(S)\left[100 + \frac{10^6}{20(S)}\right] = 9$$

$$I(S) = \frac{9}{\left[100 + \frac{10^6}{20(S)}\right]}$$

$$I(S) = \left[\frac{9}{\frac{20(S)\times 100 + 10^6}{20(S)}}\right]$$

$$I(S) = 100\, i(S) + 5\times10^4\left[\frac{I(S)}{S}\right] = \frac{45}{S}$$

$$I(S)\left[100 + \frac{5\times10^4}{S}\right] = 45/S$$

$$I(S) = \frac{45/S}{100 + \frac{5\times10^4}{S}}$$

$$I(S) = \frac{45/\cancel{S}}{\frac{100S + 5\times10^4}{\cancel{S}}} = \frac{45}{100S + 5\times10^4}$$

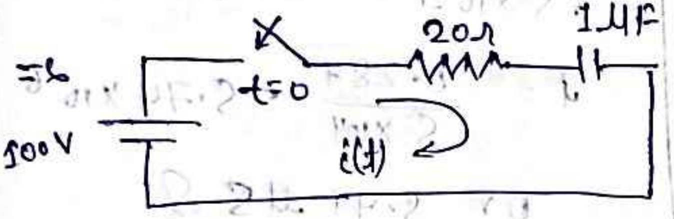
$$I(S) = \frac{45}{100\left(S + \frac{5\times10^4}{100}\right)}$$

$$I(s) = \frac{0.45}{(s+500)}$$

By applying laplace inverse on b. s.

$$\boxed{i(t) = 0.45\, e^{-500t}}$$

* For the circuit shown in fig. find the time, when the voltage across the capacitor becomes 25 Volts after the switch is closed at $t=0$.



$$\Rightarrow 20i + \frac{1}{1\times10^{-6}}\int i\,dt = 100V$$

Applying LT on both side

$$20\,Is + 10^{6}\left[\frac{I(s)}{s}\right] = \frac{100}{s}$$

$$Is\left(20 + \frac{10^6}{s}\right) = \frac{100}{s}$$

$$Is = \frac{100/s}{20 + \frac{10^6}{s}}, \quad I(s) = \frac{100/s}{\frac{20s + 10^6}{s}}$$

$$I(s) = \frac{100}{20s + 10^6}, \quad I(s) = \frac{100^{5}}{20\left(s + \frac{10^6}{20}\right)}$$

$$I(s) = \frac{5}{s + 5\times10^4}$$

$$\boxed{I(t) = 5\times e^{-5\times10^4 t}}$$

$e_R + e_C = 100$

$e_R = i(t)\cdot R$

$= 20 \times (5 \times e^{-5\times10^4 t})$

$e_R = 100 e^{-5\times10^4 t}$

Give $e_C = 25V$

$e_R + e_C = 100$

$e_R = i(t)\cdot R$

$e_R = (5 \times e^{-5\times10^4 t})\, 20$

$100 \times e^{-5\times10^4 t} + 25 = 100$

$100 \times e^{-5\times10^4 t} = 75$

$e^{-5\times10^4 t} = 0.75$

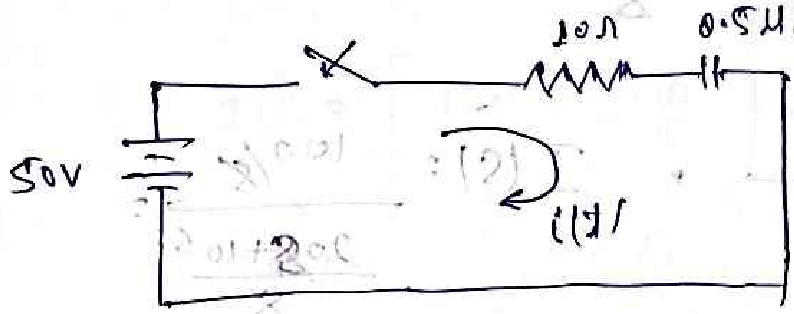
Applying log on b.s

$-5\times10^4 t = -0.287$

$t = \frac{0.287}{5\times10^4} = 5.74\times10^{-6}$

or $5.74\ \mu s$

* Find the time when the voltage across the capacitor is 10V for the given circuit.



By applying KVL

$10 I + \frac{1\times10^6}{0.5}\int i\,dt = 50$

$\frac{10^6}{0.5} \Rightarrow 10^5 \times 10$

$10 I + 2\times10^6 \int I\,dt = 50$

By applying l.t on b.s

$$10[I(S)] + 2\times10^6\left[\frac{I(S)}{S}\right] = \frac{50}{S}$$

$$I(S)\left[10 + \frac{2\times10^6}{S}\right] = \frac{50}{S}$$

$$I(S)\left[\frac{10S+2\times10^6}{S}\right] = \frac{50}{S}$$

$$I(S) = \frac{50/S}{\frac{10S+2\times10^6}{S}}$$

$e_C \times 10 = 50$

e_R

$$I(S) = \frac{50}{10S+2\times10^6}$$

$$I(S) = \frac{5}{\left(S+\frac{2\times10^6}{10}\right)}$$

L.I on b.s

$$I(t) = 5\times e^{-2\times10^5 t}$$

$$e_R + e_C = 100$$

$$e_R = i(t)\cdot R$$

$$e_R = 5\times e^{-2\times10^6}\cdot 10$$

$e^{-Q.L}$

$$\boxed{e_R = 50e^{-2\times10^5 t}}$$

$$50 \times e^{-2\times10^5 t} + 10 = 50$$

$$50 \times e^{-2\times10^5 t} = 40$$

$$e^{-2\times10^5 t} = 0.8$$

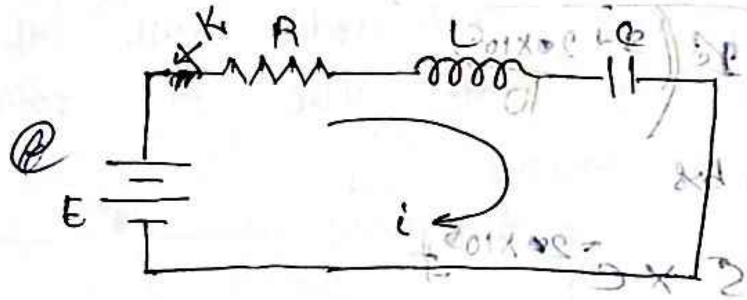
apply log on both side.

$$2\times10^5 t = 0.223$$

$$t = \frac{0.223}{2\times10^5} = 1.115\times10^{-6}$$ $\boxed{1.115\ \mu s}$

~~1.115~~ μs

* Series RLC Transient :-



Let us consider the element Resistor (R), inductor (L) and capacitor (C) which are connected in series and dc voltage E is applied through a switch K.

By applying KVL for the circuit.

$$R\cdot i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E$$

Taking L.T on both side

$$R \cdot I(S) + L\left[s(I(S)) - i(0)\right] + \frac{1}{C}\left[\frac{I(s)}{s}\right] = \frac{E}{s}$$

By applying initial conditions $i(0) = 0$

$$R I(S) + L \cdot S I(s) + \frac{1}{C(s)} I(s) = \frac{E}{s}$$

$$I s\left[R + L \cdot S + \frac{1}{Cs}\right] = \frac{E}{s}$$

$$I(S) = \frac{E/s}{R + Ls + \frac{1}{Cs}}$$

$$I(S) = \frac{E/s}{\frac{RCs + LCs^2 + 1}{Cs}} = \boxed{I(S) = \frac{EC}{RCS + LCS^2 + 1}}$$

$$I(s) = \frac{EC}{LC\left[s^2 + \frac{RC}{LC}s + \frac{1}{LC}\right]}$$

$2.\sqrt{x}$

$$\boxed{I(s) = \frac{E/L}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}}$$

$\sqrt{4x}$

$$s = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}}{2 \cdot 1}$$

$$-\frac{R}{2L} \pm \sqrt{\frac{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}{2}}$$

$$-\frac{R}{2L} \pm \sqrt{\frac{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}{4}}$$

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\pm\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{4}{LC/4}}$$

$$\alpha = \frac{-R}{2L}, \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$s = \alpha \pm \beta$$

$$s_1 = \alpha + \beta \qquad s_2 = \alpha - \beta$$

Case :- I :- Discriminant positive,

$$\boxed{\left(\frac{R}{2L}\right)^2 > \frac{1}{\sqrt{LC}}}$$

In this case we are assuming the total terms inside the root is positive then the two roots are real and positive

$$\boxed{\left(\frac{R}{2L}\right)^2 > \frac{1}{\sqrt{LC}}}$$

$$\boxed{I(s) = \frac{K_1}{(s-(\alpha+\beta))} + \frac{K_2}{(s-(\alpha-\beta))}}$$

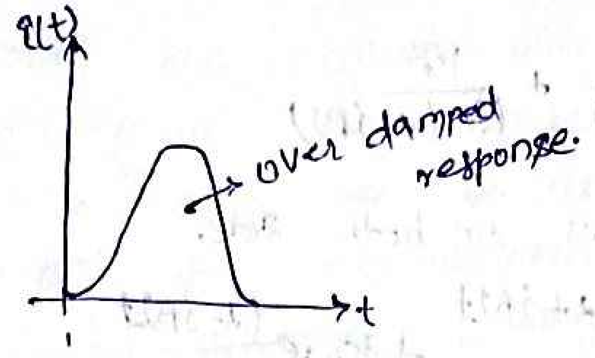
By applying I.L.T on both side,

$$i(t) = K_1 \cdot e^{(\alpha+\beta)t} + K_2 e^{(\alpha-\beta)t}$$

$$K_1 e^{\alpha t} e^{\beta t} + K_2 e^{\alpha t} e^{-\beta t}$$

$$i(t) = e^{\alpha t}\left[K_1 e^{\beta t} + K_2 e^{-\beta t}\right]$$

$$i(t) = e^{\alpha t}\left[K_1 e^{\beta t} + K_2 e^{-\beta t}\right]$$



This condition is called over damped condition. Hence the current i(t) is over damped current.

Case-II
Discriminat is zero :- In this condition $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

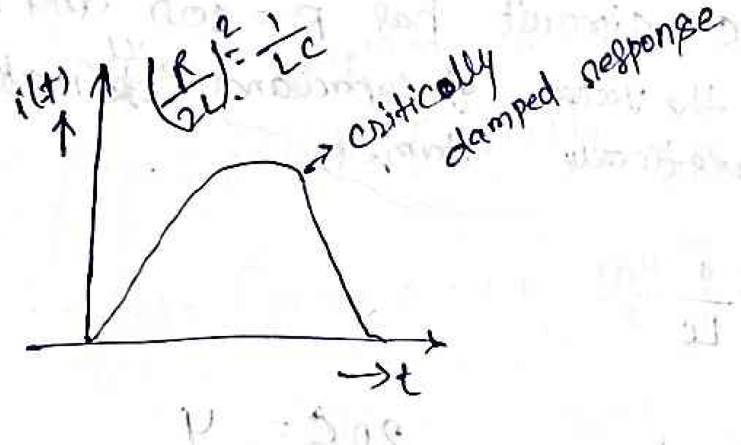
when

$$\boxed{I(S) = \frac{K_1}{(S-\alpha)} + \frac{K_2}{(S-\alpha)^2}} \qquad \boxed{L^{-1}\left[\frac{1}{(S-\alpha)^2}\right] = t \cdot e^{\alpha t}}$$

I.L.T on both sides

~~$i(t) = K_1 (S-\alpha)^2 + K_2 (S-\alpha)$~~

$$i(t) = K_1 e^{\alpha t} + K_2 \cdot t \cdot e^{\alpha t} = e^{\alpha t}\left[K_1 + t \cdot K_2\right]$$



Case III :- Discriment negative :- $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Hence the roots are complex conjugate.

these roots are $(\alpha + j\beta), (\alpha - j\beta)$

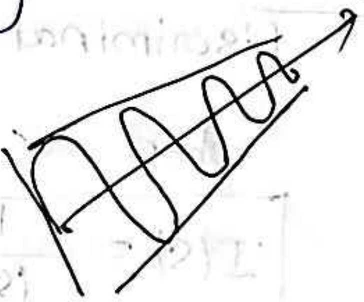
$$I(s) = \frac{K_1}{(s-(\alpha+j\beta))} + \frac{K_2}{(s-(\alpha-j\beta))}$$

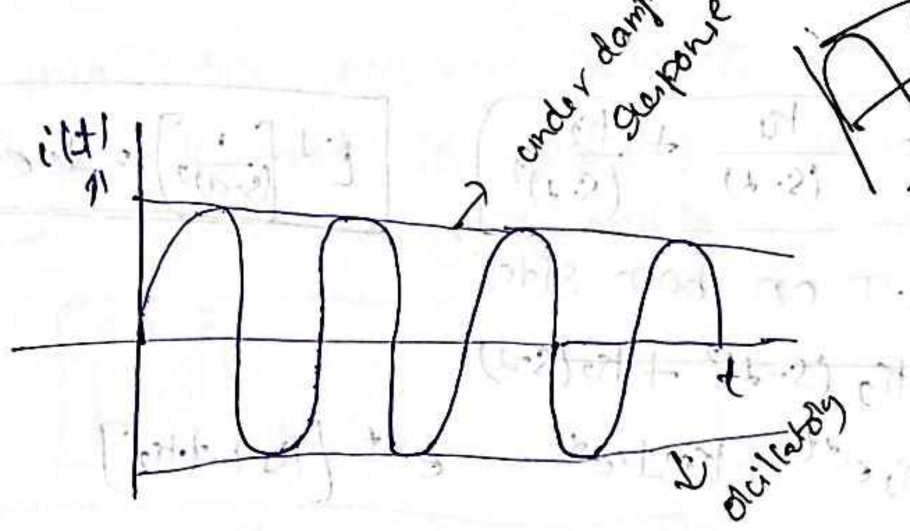
Applying L.g.T on both side,

$$i(t) = K_1 e^{(\alpha+j\beta)t} + K_2 \cdot e^{(\alpha-j\beta)t}$$

$$i(t) = K_1 \cdot e^{\alpha t} \cdot e^{j\beta t} + K_2 e^{\alpha t} \cdot e^{-j\beta t}$$

$$i(t) = e^{\alpha t}\left[K_1 \cdot e^{j\beta t} + K_2 \cdot e^{-j\beta t}\right]$$





* A Series RLC circuit has R = 10Ω and L = 2H what is the value of capacitance will make the circuit creatically damed.

=> $$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$\left(\frac{10}{2\times2}\right)^2 = \frac{1}{2\times C} = \quad 20C = 4$$

$$C = \frac{4}{20} = 0.2$$

$$\frac{\overset{25}{\cancel{100}}}{4\times \cancel{4}} = \frac{1}{2c} = \quad C = \frac{4}{50} =$$

$$C = 0.08\,F$$

* A Series RLC Circuit with $R=100\Omega$, $L=0.1$ H and $C=100\mu F$ Has a dc Voltage of 200V applied to each ~~dc to~~ at $t=0$ through a Switch find the expression for transient current by assuming initially relaxed circuit condition.

$\boxed{\text{initially relaxed means } s\,(I(0)) \text{ or } V(0)=0}$

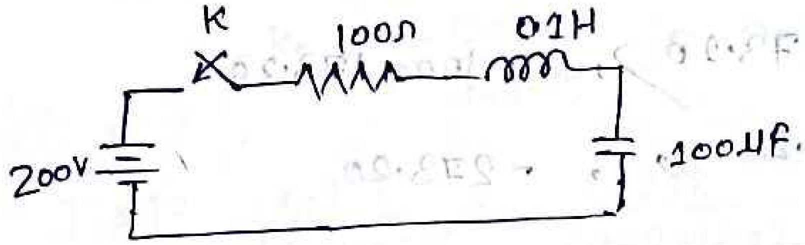
⇒ Given data,

$R=100\Omega$

$L=0.1H$

$C=100\mu f = 100\times10^{-6}$ F

Voltage = 20V



By applying KVL

$$=100I+0.1\frac{di}{dt}+\frac{1}{100\times10^{-6}}\int i\,dt = 200$$

Taking Laplace transform on both sides,

$$100\,I(s)+0.1\left[s\,I(s)\right]+\frac{1}{100\times10^{-6}}\left[\frac{I(s)}{(s)}\right]=\frac{200}{s}$$

$$I(s)=\left[100+0.1s+\frac{10^4}{s}\right]=\frac{200}{s}$$

$$I(s)=\frac{\frac{200}{s}}{\left(100+0.1s+\frac{10^4}{s}\right)}\qquad ;\ I(s)=\frac{\frac{200}{\cancel{s}}}{\frac{100s+0.1s^2+10^4}{\cancel{s}}}$$

$$I(s) = \frac{200}{(0.1s^2+100s+10^4)}$$

$$0.1s^2+100s+10^4=0$$

$$s = \frac{-100 \pm \sqrt{(100)^2-4(0.1)(10^4)}}{2\times 0.1}$$

$$\cancel{s = -100 \pm \sqrt{\frac{10000-4000}{2\times 0.1}}}$$

$$\cancel{s \quad -100 \pm 173.20}$$

$$\cancel{s = -100+173.20}, \; \cancel{-100-173.20}$$

$$\cancel{s \quad = 73.2, \; -273.20}$$

$$s = \frac{-100 \pm \sqrt{10000-0.4\times 10^4}}{2\times 0.1}$$

$$s = \frac{-100 \pm \sqrt{6000}}{2\times 0.1}$$

$$s = \frac{-100 \pm 20\sqrt{15}}{0.2}$$

$$s = -112.70, \quad s = -887.29$$

$$I(s) = \frac{200}{(s+112.70)(s+887.29)}$$

$$I(s) = \frac{200}{(s+112.70)(s+887.29)} = \frac{K_1}{(s+112.70)} + \frac{K_2}{(s+887.29)}$$

$$200 = K_1(s+887.29) + K_2(s+112.70)$$

Put $s = -112.70$

then $\boxed{K_1 = 0.258}$

Put $s = -887.29$

$$200 = 0 + K_2(-887.29 + 112.70)$$

$$200 = K_2(-774.59)$$

$$K_2 = \frac{-200}{774.59} = -0.258$$

$$I(s) = \frac{0.258}{(s+112.70)} - \frac{0.258}{(s+887.29)}$$

By applying I.L.T on both sides.

$$i(t) = 0.258\, e^{-112.70t} - 0.258\, e^{-887.29t}$$

* A Series RLC Circuit with $R = 300\Omega$, $L = 1\,H$, $C = 100\mu F$ has a Constent Voltage of 50V applied to it at $t=0$, Find the maximum current value by assuming zero initial condition

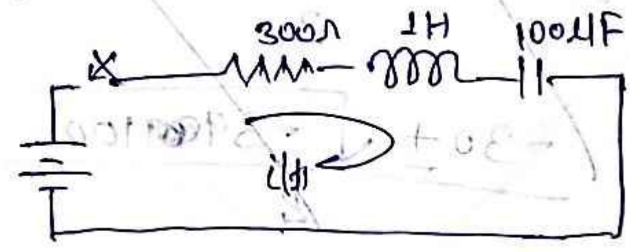
=> Given that

$R = 300\Omega$

$L = 1\,H$

$C = 100 \times 10^{-6} F$

$V = 50\,V$



By applying KVL,

$$300(I) + 1\left(\frac{d(i)}{d(t)}\right) + \frac{1}{100\times10^{-6}}\int(i \cdot dt) = 50$$

Lapalace trans on b.s

$$300I(S) + 1[s(I(S))] + \frac{1}{100\times10^{-6}}\left[\frac{I(s)}{(s)}\right] = \frac{50}{s}$$

$$I(S)\left[300 + s + \frac{100^{-2}\times10^{6}}{s}\right] = \frac{50}{s}$$

$$I(S) = \frac{\frac{50}{s}}{\frac{300s + s^2 + 100\times10^4}{s}}$$

$$I(S) = \frac{50}{300s + s^2 + 100\times10^4}$$

$a = 1$

$b = 300$

$c = 100\times10^4$

$$= \frac{-30 \pm \sqrt{(300)^2 - 4(1)(100\times10^4)}}{2\times1}$$

$s = 96.80$

$$= \frac{-30 \pm \sqrt{900 - 4\times10^4}}{2}$$

$$\frac{-30 \pm \sqrt{-39100}}{2}$$

$$S = \frac{-300 \pm \sqrt{(300)^2 - 4(1)(00\times10^4)}}{2\times 1}$$

$$S = \frac{-300 \pm \sqrt{90000 - 4\times10^4}}{2}$$

$$S = \frac{-300 \pm 223.60}{2}$$

$$S = \frac{-300 + 223.60}{2}, \quad S = \frac{-300 - 223.60}{2}$$

$$S = -38.2, \quad S = -261.8$$

$$I(S) = \frac{50}{(S+38.2)(S+26.18)} = \frac{K_1}{(S+38.2)} + \frac{K_2}{(S+26.18)}$$

$$50 = K_1(S+26.18) + K_2(S+38.2)$$

Put : $S = -26.18$

$$50 = K_2(-26.18 + 38.2), \quad K_2 = \frac{50}{12.02} = 4.159$$

Put $S = -38.2$

$$50 = (-38.2 + 26.18) + K_2(0)$$

$50 =$

* A.C Transients :-

(i) Series R-L Circuit.

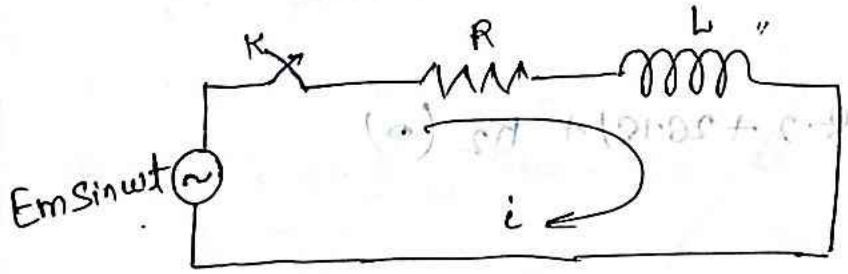
(ii) Series R-C Circuit

(iii) Series R-L-C Circuits

* Series R-L Circuit:- Let us Consider a resistor (R) and Inductor (L) which are Connected in series the applied Voltage is AC Voltage that is $E_m \sin\omega t$. The AC Voltage is connected to the Series Circuit through a switch K.

Let us assumed the switch is closed at $t=0$, the initial current flowing through the inductor is assumed as zero.

Then by applying KVL for the circuit.



By applying KVL for the circuit.

$$iR + L\frac{di}{dt} = E_m \sin\omega t$$

Taking LT on both sides. LT $\sin at = \frac{a}{s^2+a^2}$

$$\boxed{L^{-1} I(s) = \frac{a}{s^2+a^2}}$$

$$I(s)R + L\left[sI(s)\right] = \frac{E_m \cdot \omega}{s^2+\omega^2}$$

$$I(s)\left[R + L(s)\right] = \frac{E_m \cdot \omega}{s^2+\omega^2}$$

$$\cancel{R+L(s)} \quad I(s) = \frac{E_m \cdot \omega}{(s^2+\omega^2)(R+L(s))}$$

$$\boxed{I(s) = \frac{E_m \cdot \omega}{(s^2+\omega^2) \cdot L\left(s+\frac{R}{L}\right)}}$$

$$= \frac{E_m\ \omega/L}{(s^2+\omega^2)\left(s+\frac{R}{L}\right)}$$

$$\boxed{I(s) = \frac{\frac{E_m \omega}{L}}{(s+j\omega)(s-j\omega)\left(s+\frac{R}{L}\right)}}$$

$$B = \frac{1}{2j\omega\left(\frac{j\omega L+R}{L}\right)}$$

$$\boxed{I(s) = \frac{\frac{E_m \omega}{L}}{(s+j\omega)(s-j\omega)\left(s+\frac{R}{L}\right)} = \frac{A}{(s+j\omega)} + \frac{B}{(s-j\omega)} + \frac{C}{\left(s+\frac{R}{L}\right)}} \quad \text{(i)}$$

$$\boxed{\frac{E_m \omega}{L} = A(s-j\omega)\left(s+\frac{R}{L}\right) + B(s+j\omega)\left(s+\frac{R}{L}\right) + C(s+j\omega)(s-j\omega)} \quad \text{(ii)}$$

Put $s = -j\omega$ in equ (ii)

$$\frac{E_m \omega}{L} = A \times (-2j\omega)\left(-j\omega + \frac{R}{L}\right)$$

$A = 2j\omega$

$$A = \frac{\frac{E_m \omega}{L}}{-2j\omega\left(-j\omega + \frac{R}{L}\right)}$$

$$A = \frac{\frac{E_m \omega}{L}}{\left(2j^2\omega^2 - \frac{2j\omega R}{L}\right)}$$

$$A = \frac{E_m\omega/L}{-2\omega^2 - j\frac{2\omega R}{L}} = \frac{\frac{E_m\omega}{L}}{\frac{2\omega}{L}\left[-\omega L - jR\right]}$$

$$\cancel{A = \frac{E_m\omega}{L}} \qquad A = \frac{E_m}{2(-\omega L - jR)}$$

$$A = \frac{E_m}{-2(\omega L + jR)} \times \frac{\omega L - jR}{\omega L - jR}$$

$$A = \frac{-E_m(\omega L - jR)}{2(\omega^2 L^2 + R^2)}$$

Put $S = j\omega$ in equ (i)

$$\frac{E_m\omega}{L} = 0 + B(2j\omega)\left(\frac{LS+R}{L}\right) + 0$$

$$\frac{E_m\omega}{L} = B(2j\omega)\left(\cancel{\frac{LS+R}{L}}\right)\left(j\omega + \frac{R}{L}\right)$$

$$B = \frac{\frac{E_m\omega}{L}}{(2j\omega)\left(\frac{j\omega L + R}{L}\right)}$$

$$B = \frac{\frac{Em\omega}{L}}{\frac{2j^2\omega^2 L + 2j\omega R}{L}}$$

$$B = \frac{Em\omega}{-2\omega^2 L + 2j\omega R} = \frac{Em\omega}{\omega(-2\omega L + 2jR)}$$

$$B = \frac{-Em}{2\omega L - 2jR} = \frac{-Em}{2(\omega L - jR)}$$

ωL

$$\frac{-Em}{2(\omega L - jR)} \times \frac{\omega L + jR}{\omega L + jR}$$

$$= \frac{-Em(\omega L + jR)}{2(\omega^2 L^2 + R^2)}$$

$$\boxed{\text{Put } S = -\frac{R}{L}} \text{ in equ (ii)}$$

$$\frac{Em\omega}{L} = C\left(-\frac{R}{L} + j\omega\right)\left(-\frac{R}{L} - j\omega\right)$$

$$C = \frac{\frac{Em\omega}{L}}{\left(j\omega - \frac{R}{L}\right)\left(-j\omega - \frac{R}{L}\right)}$$

$$C = \frac{\frac{Em\omega}{L}}{-\left[\left(\frac{R}{L} - j\omega\right)\left(\frac{R}{L} + j\omega\right)\right]}$$

$$C = \frac{\cancel{+} Em\omega / L}{\frac{R^2}{L^2} + \omega^2}$$

$$C = \frac{\frac{\cancel{+} Em\omega}{L}}{\frac{R^2 + \omega^2 L^2}{L^2}}$$

$$C = \frac{\cancel{+} Em\omega \times L}{R^2 + \omega^2 L^2} = \frac{Em\,\omega L}{(R^2 + \omega^2 L^2)}$$

$$C = \frac{\cancel{+} Em\omega \cdot \times \cancel{L}}{\cancel{L}\left(\frac{R^2}{L} \cancel{+} \omega^2 L\right)}$$

$$\Rightarrow \frac{E_m \cdot \omega \cdot \times L}{L\left(\frac{R^2}{L} + \omega^2 L\right)}$$

Sub the Value of A, B & C in equ (i)

$$I(S) = \frac{\frac{-Em(\omega L - jR)}{2(\omega^2L^2 + R^2)}}{S + j\omega} - \frac{\frac{Em(\omega L + jR)}{2(\omega^2L^2 + R^2)}}{S - j\omega} + \frac{\frac{Em\omega L}{\omega^2 L^2 + R^2}}{\left(S + \frac{R}{L}\right)}$$

Taking Laplace inverse on both sides.

$$i(t) = \left[\frac{Em\omega L}{\omega^2 L^2 + R^2}\right] \times e^{\frac{-R}{L}t} - \frac{Em(\omega L - jR)}{2(\omega^2 L^2 + R^2)}\left[e^{-j\omega t}\right] - \frac{Em(\omega L + jR)}{2(\omega^2 L^2 + R^2)} e^{j\omega t}$$

$$i(t) = \frac{E_m \omega L}{\omega^2 L^2 + R^2} e^{-R/L\, t} - \frac{E_m}{2(\omega^2 L^2 + R^2)} \left[e^{-j\omega t}(\omega L - jR) + e^{j\omega t}(\omega L + jR) \right]$$

$$\boxed{\begin{aligned} e^{j\theta} &= \cos\theta + j\sin\theta \\ e^{-j\theta} &= \cos\theta - j\sin\theta \end{aligned}}$$

$$i(t) = \frac{E_m \omega L}{\omega^2 L^2 + R^2} e^{-R/L\, t} - \frac{E_m}{2(\omega^2 L^2 + R^2)} \left[(\cos\omega t - j\sin\omega t)(\omega L - jR) + (\cos\omega t + j\sin\omega t)(\omega L + jR) \right]$$

$$i(t) = \frac{E_m \omega L}{\omega^2 L^2 + R^2} e^{-R/L\, t} - \frac{E_m}{2(\omega^2 L^2 + R^2)} \left[(\cos\omega t)\omega L - \cancel{jR\cos\omega t} - \omega L \cancel{j\sin\omega t} - R\sin\omega t + \omega L\cos\omega t + \cancel{\omega L j\sin\omega t} + \cancel{jR\cos\omega t} - R\sin\omega t \right]$$

$$i(t) = \frac{E_m \omega L}{\omega^2 L^2 + R^2} e^{-R/L\, t} - \frac{E_m}{2(\omega^2 L^2 + R^2)} \left[2\omega L\cos\omega t - 2R\sin\omega t \right]$$

$$i(t) = \frac{E_m \omega L}{\omega^2 L^2 + R^2} e^{-R/L\, t} - \frac{E_m \cancel{2}}{\cancel{2}(\omega^2 L^2 + R^2)} \left[\omega L\cos\omega t - R\sin\omega t \right]$$

$$i(t) = \frac{E_m}{\omega^2 L^2 + R^2} \left[e^{-R/L\, t}\omega L - \omega L\cos\omega t + R\sin\omega t \right]$$

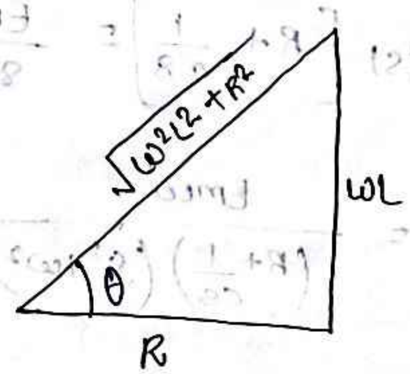
$$\boxed{\omega^2 L^2 + R^2 = \left(\sqrt{\omega^2 L^2 + R^2}\right)\left(\sqrt{\omega^2 L^2 + R^2}\right)}$$

$$i(t) = \frac{E_m}{\sqrt{\omega^2 L^2 + R^2}} \left[\frac{e^{-\frac{R}{L}t}\omega L}{\sqrt{R^2 + \omega^2 L^2}} - \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}\cos\omega t + \frac{R}{\sqrt{\omega^2 L^2 + R^2}}\sin\omega t \right]$$

From Triangle

$$\sin\theta = \frac{\omega L}{\sqrt{\omega^2 L^2 + R^2}}$$

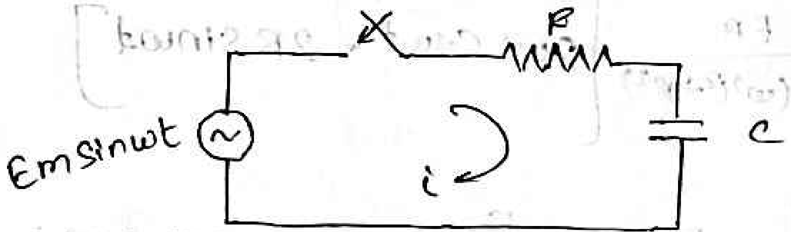
$$\cos\theta = \frac{R}{\sqrt{\omega^2 L^2 + R^2}}$$



$$i(t) = \frac{Em}{\sqrt{\omega^2L^2+R^2}}\left[e^{-\frac{R}{L}t}\sin\theta - \cos\omega t \cdot \sin\theta + \cos\theta \cdot \sin\omega t\right]$$

$$\boxed{i(t) = \frac{Em}{\sqrt{\omega^2L^2+R^2}}\left[e^{-\frac{R}{L}t}\sin\theta + \sin(\omega t-\theta)\right]}$$

RC transients :- Let us consider a series RC circuit which is energies by AC supply using switch K. Assuming initially relaxed ~~capa~~ condition then charge across the capacitor is zero.



By applying KVL for the circuit,

$$R \cdot i + \frac{1}{C}\int i \cdot dt = Em \sin\omega t$$

Taking laplace transform on both sides,

$$R \cdot I(s) + \frac{1}{C} \cdot \frac{I(s)}{s} = \frac{Em\omega}{s^2+\omega^2}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{Em\omega}{s^2+\omega^2}$$

$$I(s) = \frac{Em\omega}{\left(R+\frac{1}{Cs}\right)\left(s^2+\omega^2\right)}$$

$$I(s) = \frac{Em\omega}{\frac{1}{Cs}\left[RCs+1\right]\left[s^2+\omega^2\right]}$$

$$= \frac{Em\omega \times Cs}{(RCs+1)(s^2+\omega^2)}$$

$$I(s) = \frac{Em\omega\, Cs}{RC\left(s+\frac{1}{RC}\right)(s^2+\omega^2)}, \quad \frac{Em\omega\, s/R}{\left(s+\frac{1}{RC}\right)(s+j\omega)(s-j\omega)}$$

$$\frac{Em\omega s/R}{\left(s+\frac{1}{RC}\right)(s+j\omega)(s-j\omega)} = \frac{A}{s+\frac{1}{RC}} + \frac{B}{s+j\omega} + \frac{C}{s-j\omega} \quad \text{---} \quad (a)$$

$$\text{(1)} - \boxed{\frac{Em\omega s}{R} = A(s+j\omega)(s-j\omega) + B\left(s+\frac{1}{RC}\right)(s-j\omega) + C\left(s+\frac{1}{RC}\right)(s+j\omega)}$$

Put $s = -\frac{1}{RC}$ in equ (1)

$$\frac{Em\omega s}{R} = A\left(\frac{-1}{RC}+j\omega\right)\left(\frac{-1}{RC}-j\omega\right) + 0 + 0$$

$$\frac{Em\omega s}{R} = A\left(\frac{1}{RC}-j\omega\right)\left(\frac{1}{RC}+j\omega\right)$$

$$\frac{Em\omega s}{R} = A\left[\left(\frac{1}{RC}\right)^2 + \omega^2\right]$$

$$A = \frac{\frac{Em\omega s}{R}}{\left[\frac{1}{R^2C^2} + \frac{\omega^2}{1}\right]}, \quad A = \frac{\frac{Em\omega s}{R}}{\frac{1+R^2C^2\omega^2}{R^2C^2}}$$

$$\boxed{A = \frac{Em\omega s RC^2}{(1+R^2C^2\omega^2)}}$$

$s = -1/RC$

$$A = \frac{Em\omega \times \frac{-1}{RC} \times RC^2}{(1+R^2C^2\omega^2)}$$

$$\boxed{A = \frac{-Em\omega C}{1+\omega^2R^2C^2}}$$

Put $s = -j\omega$ in (i)

$$\frac{E_m\omega s}{R} = 0 + B\left(-j\omega + \frac{1}{RC}\right)\left(-j\omega - j\omega\right) + 0$$

$$\frac{E_m\omega s}{R} = B\left[j\omega - \frac{1}{RC}\right]\left[j\omega + j\omega\right]$$

$$\frac{E_m\omega s}{R} = B\left[(j\omega)^2 - \left(\frac{1}{RC}\right)^2\right] \qquad \frac{E_m\omega s}{R} = B\left[j\omega - \frac{1}{RC}\right]\left[2j\omega\right]$$

$$\frac{E_m\omega \times -j\omega}{R} = B\left[-\omega^2 - \frac{1}{R^2C^2}\right] \qquad \frac{E_m\omega s}{R} = B\left[\frac{j\omega RC - 1}{RC}\right](2j\omega)$$

$$\frac{-E_m\omega j\omega}{R} = B\left[\frac{-(R^2C^2\omega^2) - 1}{R^2C^2}\right] \qquad B = \frac{\frac{E_m\omega s}{R}}{\left(\frac{j\omega RC - 1}{(RC)}\right)(2j\omega)}$$

$$\frac{\frac{-E_m\omega j\omega}{R}}{\frac{-R^2C^2\omega^2 - 1}{R^2C^2}} = \frac{\frac{\cancel{-}E_m\omega j\omega}{R}}{\frac{\cancel{-}(R^2C^2\omega^2 + 1)}{R^2C^2}} \qquad B = \frac{\frac{E_m\omega(-j\omega)}{R}}{\left(\frac{j\omega RC - 1}{RC}\right)(2j\omega)}$$

$$B = \frac{E_m\omega j\omega}{R} \times \frac{R^2C^2}{R^2C^2\omega^2 + 1} \qquad B = \frac{\frac{-E_m\omega}{R}}{\frac{j\omega^2 RC - j\omega}{RC}}$$

$$B = \frac{\frac{\cancel{-}E_m\omega}{R}}{\frac{\cancel{-}[\omega^2 RC + j\omega)}{RC}} \qquad B = \frac{E_m\omega C}{\omega^2 RC + j\omega}$$

$$B = \frac{\frac{E_m\omega \cdot (-\cancel{j\phi})}{R}}{\frac{j\omega RC - 1}{RC} \times 2\cancel{j\phi}}$$

$$(C) = \frac{-\frac{E_m\omega}{R}}{2 \times \frac{j\omega RC - 1}{RC}}$$

$$= \frac{E_m\omega C}{2(1 - j\omega RC)} \times (1 + j\omega RC)$$

$$= \frac{E_m\omega C}{2(1 - j\omega RC)} \times \frac{1 + j\omega RC}{1 + j\omega RC}$$

$$B = \frac{E_m\omega C\,(1 + j\omega RC)}{2\,(1 + \omega^2 R^2 C^2)}$$

~~Put C =~~ Put $s = j\omega$ in equ (1)

$$\frac{E_m\omega(j\omega)}{R} = 0 + 0 + C\left(j\omega + \frac{1}{RC}\right)(j\omega + j\omega)$$

$$\frac{E_m\omega(j\omega)}{R} = C\left[\frac{j\omega RC + 1}{RC}\right]\left[2j\omega\right]$$

$$\frac{E_m\omega(\cancel{j\omega})}{R\left[\frac{j\omega RC + 1}{RC}\right]\left[2\cancel{j\omega}\right]} = \boxed{\frac{E_m\omega/R}{2\left[\frac{j\omega RC + 1}{RC}\right]} = C}$$